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With Great Power . . .
Inverses of Power Functions

Vocabulary

Write a definition for each term in your own words.

1. inverse of a function

   The **inverse of a function** is the set of all ordered pairs (y, x), or $(f(x), x)$.

2. invertible function

   If the inverse of a function f is also a function, then f is an **invertible function**, and its inverse is written as $f^{-1}(x)$.

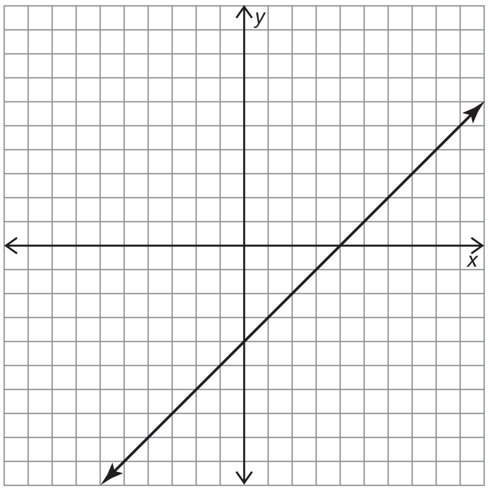
3. Horizontal Line Test

   A **Horizontal Line Test** is used to determine whether a function has an inverse that is also a function. That is, if a horizontal line can pass through more than one point on the graph at the same time, then the function is *not* invertible.

Problem Set

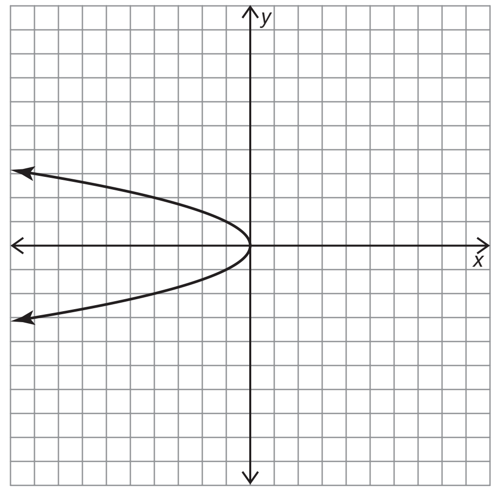
Determine whether or not each relation is a function. Use the Vertical Line Test.

1.



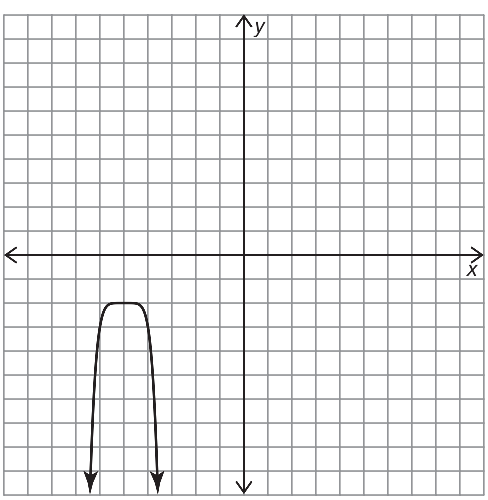
The relation is a function because it passes the Vertical Line Test.

2.



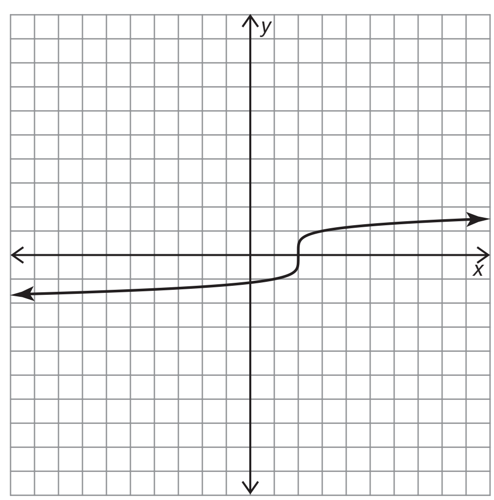
The relation is *not* a function because it does *not* pass the Vertical Line Test.

3.



The relation is a function because it passes the Vertical Line Test.

4.



The relation is a function because it passes the Vertical Line Test.

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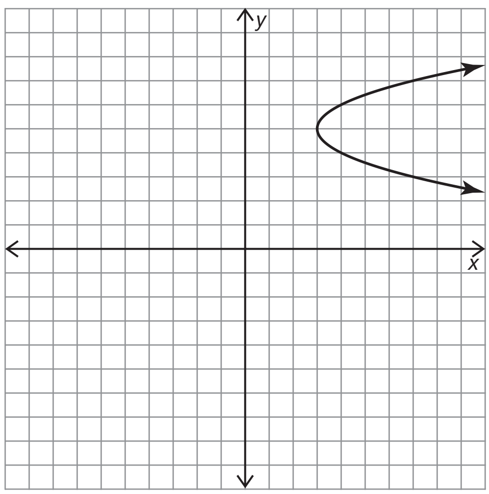
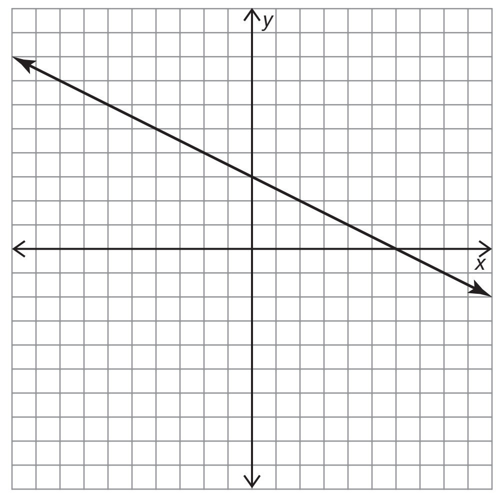
5.



The relation is *not* a function because it does *not* pass the Vertical Line Test.

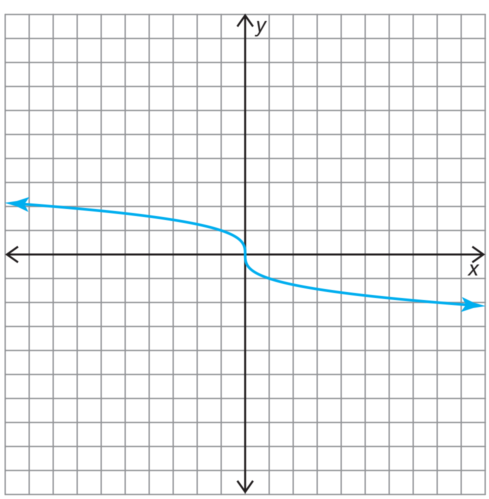
6.



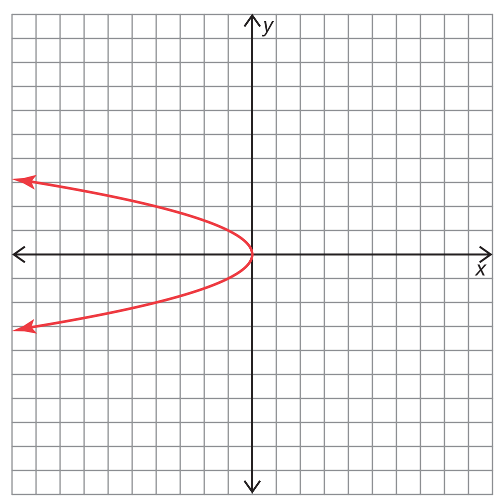
The relation is a function because it passes the Vertical Line Test.

Sketch the graph of the inverse of each function.

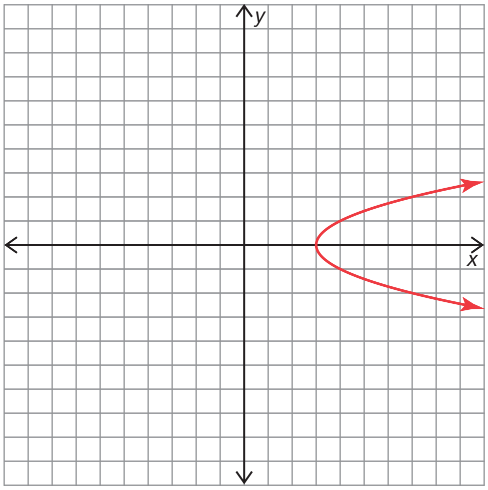
7. $y = -(x^3)$



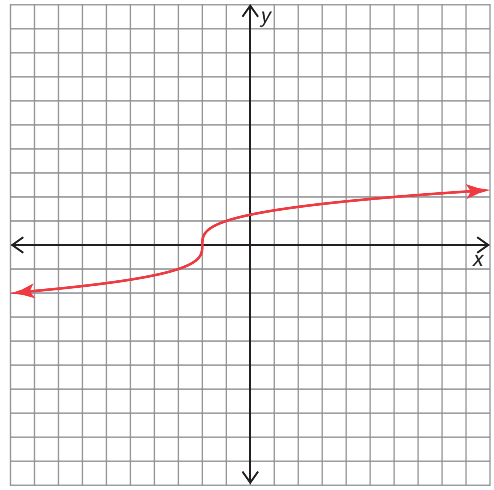
8. $y = -(x^2)$



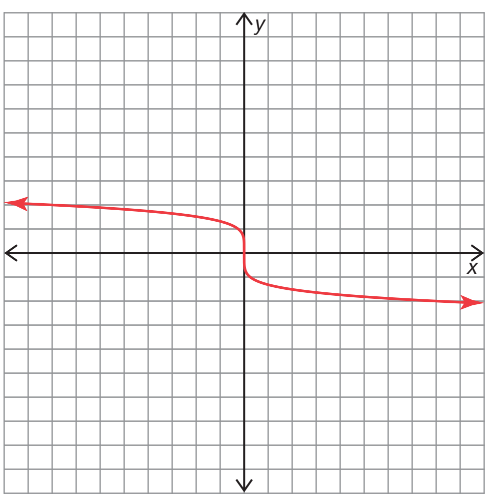
9. $y = x^2 + 3$



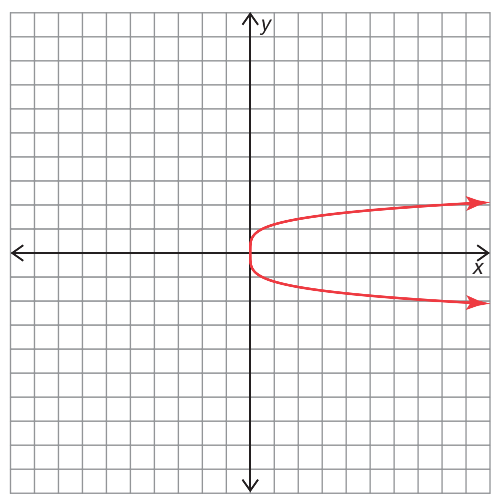
10. $y = x^3 - 2$



11. $y = -\frac{1}{4}x^5$



12. $y = \frac{1}{2}x^4$



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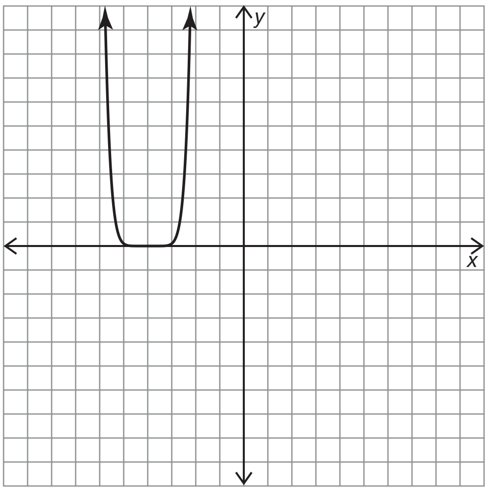
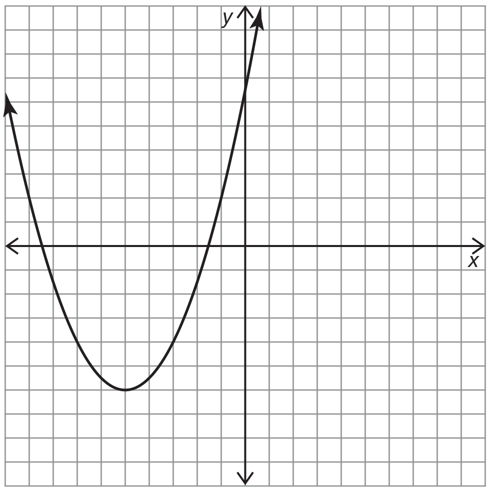
Determine whether each function is invertible. Use the Horizontal Line Test.

13.



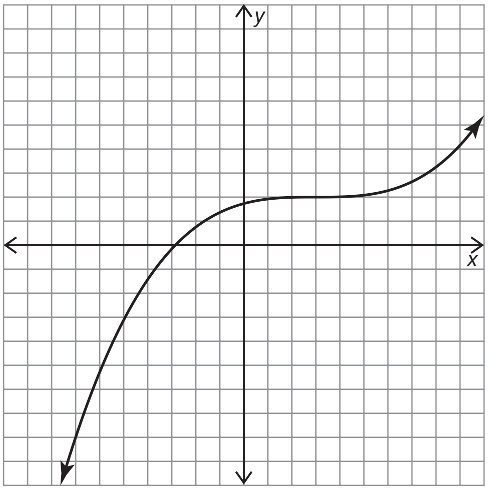
This function is *not* invertible, because it does *not* pass the Horizontal Line Test.

14.



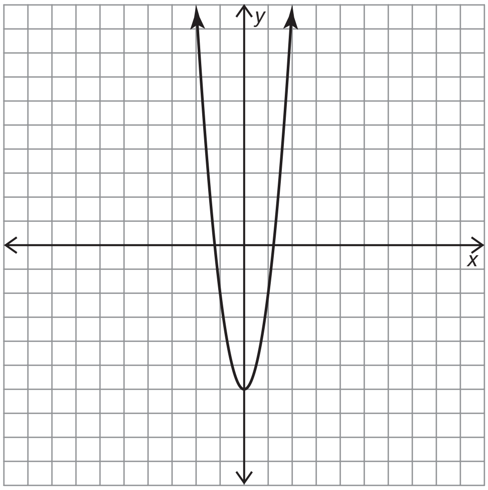
This function is *not* invertible, because it does *not* pass the Horizontal Line Test.

15.



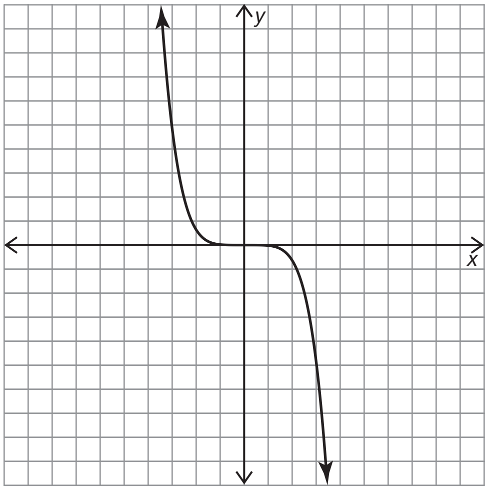
This function is invertible, because it passes the Horizontal Line Test.

16.



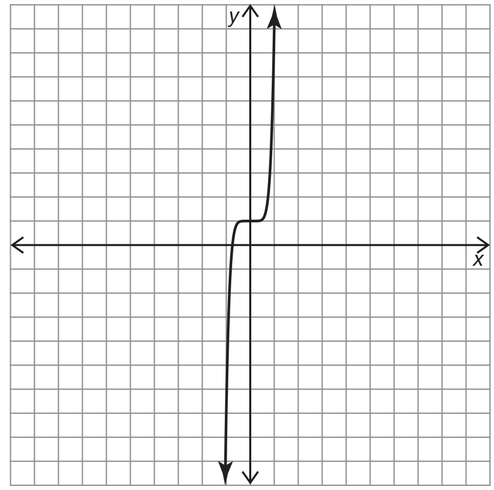
This function is *not* invertible, because it does *not* pass the Horizontal Line Test.

17.



This function is invertible, because it passes the Horizontal Line Test.

18.



This function is invertible, because it passes the Horizontal Line Test.

Without graphing, determine whether or not each function is invertible.

19. $y = 3x^2$

This function is *not* invertible, because it is an even power function.

20. $y = x^{24}$

This function is *not* invertible, because it is an even power function.

21. $y = -x^{99}$

This function is invertible, because it is an odd power function.

22. $y = 1.257x^{10}$

This function is *not* invertible, because it is an even power function.

23. $y = 2x^{15}$

This function is invertible, because it is an odd power function.

24. $y = -\frac{3}{5}x^{124}$

This function is *not* invertible, because it is an even power function.

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The Root of the Matter
Radical Functions

Vocabulary

Provide an example of each term.

1. square root function
   Answers will vary but should be of the form $f(x) = \sqrt{x}$, for $x \geq 0$.

2. cube root function
   Answers will vary but should be of the form $f(x) = \sqrt[3]{x}$.

3. radical function
   Answers will vary but should be of the form $f(x) = \sqrt[n]{x}$.

4. composition of functions
   Answers will vary but should be in the form $(f \circ g)(x)$ or $f(g(x))$.

Problem Set

Determine the equation for the inverse of each function. Show your work.

1. $f(x) = 4x^2$

$y = 4x^2$

$x = 4y^2$

$\frac{x}{4} = y^2$

$\pm\frac{\sqrt{x}}{2} = y$

2. $f(x) = \frac{2}{5}x^2$

$y = \frac{2}{5}x^2$

$x = \frac{2}{5}y^2$

$\frac{5}{2}x = y^2$

$\pm\sqrt{\frac{5}{2}x} = y$

$\pm\frac{\sqrt{10x}}{2} = y$

3. $f(x) = x^2 + 7$

$$y = x^2 + 7$$
$$x = y^2 + 7$$
$$x - 7 = y^2$$
$$\pm\sqrt{x - 7} = y$$

4. $f(x) = x^2 - 9$

$$y = x^2 - 9$$
$$x = y^2 - 9$$
$$x + 9 = y^2$$
$$\pm\sqrt{x + 9} = y$$

5. $f(x) = (x + 3)^2$

$$y = (x + 3)^2$$
$$x = (y + 3)^2$$
$$\pm\sqrt{x} = y + 3$$
$$\pm\sqrt{x} - 3 = y$$

6. $f(x) = 9x^3$

$$y = 9x^3$$
$$x = 9y^3$$
$$\frac{x}{9} = y^3$$
$$\sqrt[3]{\frac{x}{9}} = y$$
$$\frac{\sqrt[3]{x}}{3} = y$$

7. $f(x) = \frac{1}{8}x^3$

$$y = \frac{1}{8}x^3$$
$$x = \frac{1}{8}y^3$$
$$8x = y^3$$
$$2\sqrt[3]{x} = y$$

8. $f(x) = x^3 + 27$

$$y = x^3 + 27$$
$$x = y^3 + 27$$
$$x - 27 = y^3$$
$$\sqrt[3]{x - 27} = y$$

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9. $f(x) = x^3 - 6$

$y = x^3 - 6$

$x = y^3 - 6$

$x + 6 = y^3$

$\sqrt[3]{x + 6} = y$

10. $f(x) = (x - 1)^3$

$y = (x - 1)^3$

$x = (y - 1)^3$

$\sqrt[3]{x} = y - 1$

$\sqrt[3]{x} + 1 = y$

11. $f(x) = x^4$

$y = x^4$

$x = y^4$

$\pm\sqrt[4]{x} = y$

12. $f(x) = \frac{1}{32}x^5$

$y = \frac{1}{32}x^5$

$x = \frac{1}{32}y^5$

$32x = y^5$

$2\sqrt[5]{x} = y$

Identify the characteristics (domain, range, and the *x*- and *y*-intercepts) of each function.

13. $f(x) = \sqrt{3x}$

Domain: $[0, \infty)$

Range: $[0, \infty)$

x-intercept: (0, 0)

y-intercept: (0, 0)

14. $f(x) = \sqrt{x + 4}$

Domain: $[-4, \infty)$

Range: $[0, \infty)$

x-intercept: $(-4, 0)$

y-intercept: (0, 2)

15. $f(x) = \sqrt{x} + 1$

Domain: $[0, \infty)$

Range: $[1, \infty)$

x-intercept: none

y-intercept: (0, 1)

16. $f(x) = \frac{\sqrt{x}}{2}$

Domain: $[0, \infty)$

Range: $[0, \infty)$

x-intercept: (0, 0)

y-intercept: (0, 0)

17. $f(x) = \sqrt{-5x}$

Domain: $(-\infty, 0]$

Range: $[0, \infty)$

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

18. $f(x) = \sqrt{3 - x}$

Domain: $(-\infty, 3]$

Range: $[0, \infty)$

x-intercept: $(3, 0)$

y-intercept: $(0, 3)$

19. $f(x) = \sqrt[3]{4x}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

20. $f(x) = \sqrt[3]{x - 2}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(2, 0)$

y-intercept: $(0, \sqrt[3]{-2})$

21. $f(x) = \sqrt[3]{x} - 5$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $\sqrt[3]{5}, 0)$

y-intercept: $(0, -5)$

22. $f(x) = \frac{\sqrt[3]{x}}{4}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

23. $f(x) = \sqrt[3]{-2x}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

24. $f(x) = \sqrt[3]{1 - x}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x-intercept: $(1, 0)$

y-intercept: $(0, 1)$

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Use compositions to determine whether $f(x)$ and $g(x)$ are inverse functions. Show your work.

25. $f(x) = \dfrac{-8 + x}{2}$ $\qquad g(x) = 2x + 8$

The functions $f(x)$ and $g(x)$ are inverse functions because $f(g(x)) = g(f(x)) = x$.

$$f(g(x)) = \frac{-8 + (2x + 8)}{2} = \frac{2x}{2} = x$$

$$g(f(x)) = 2\left(\frac{-8 + x}{2}\right) + 8 = -8 + x + 8 = x$$

26. $f(x) = -4x + 9$ $\qquad g(x) = \dfrac{x - 4}{-9}$

The functions $f(x)$ and $g(x)$ are not inverse functions because $f(g(x)) \neq x$.

$$f(g(x)) = -4\left(\frac{x - 4}{-9}\right) + 9 = \frac{-4x + 16}{-9} + 9$$

27. $f(x) = (x - 2)^2$ $\qquad g(x) = \sqrt{x} - 2$

The functions $f(x)$ and $g(x)$ are not inverse functions because $f(g(x)) \neq x$.

$$f(g(x)) = ((\sqrt{x} - 2) - 2)^2 = (\sqrt{x} - 4)^2 = x - 8\sqrt{x} + 16$$

28. $f(x) = 5x^2$ $\qquad g(x) = \sqrt{\dfrac{x}{5}}$

The functions $f(x)$ and $g(x)$ are inverse functions because $f(g(x)) = g(f(x)) = x$.

$$f(g(x)) = 5\left(\sqrt{\frac{x}{5}}\right)^2 = 5\left(\frac{x}{5}\right) = x$$

$$g(f(x)) = \sqrt{\frac{(5x^2)}{5}} = \sqrt{x^2} = x$$

11

29. $f(x) = 2\sqrt[3]{x+3}$ $g(x) = \frac{x^3}{8} - 3$

The functions $f(x)$ and $g(x)$ are inverse functions because $f(g(x)) = g(f(x)) = x$.

$$f(g(x)) = 2\sqrt[3]{\left(\frac{x^3}{8} - 3\right) + 3}$$
$$= 2\sqrt[3]{\frac{x^3}{8}}$$
$$= 2\left(\frac{x}{2}\right)$$
$$= x$$

$$g(f(x)) = \frac{\left(2\sqrt[3]{x+3}\right)^3}{8} - 3$$
$$= \frac{8(x+3)}{8} - 3$$
$$= x + 3 - 3$$
$$= x$$

30. $f(x) = 3(x+1)^3$ $g(x) = \sqrt[3]{\frac{x}{3}} - 1$

The functions $f(x)$ and $g(x)$ are inverse functions because $f(g(x)) = g(f(x)) = x$.

$$f(g(x)) = 3\left(\left(\sqrt[3]{\frac{x}{3}} - 1\right) + 1\right)^3$$
$$= 3\left(\sqrt[3]{\frac{x}{3}}\right)^3$$
$$= 3\left(\frac{x}{3}\right)$$
$$= x$$

$$g(f(x)) = \sqrt[3]{\frac{(3(x+1)^3}{3}} - 1$$
$$= \sqrt[3]{(x+1)^3} - 1$$
$$= x + 1 - 1$$
$$= x$$

11

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Complete each exercise.

31. The distance to the horizon is given by the equation $d = \sqrt{h(D + h)}$, where h represents the height of the observer in feet and D represents the diameter of the Earth in miles. Write the equation as a function of the height and use 7918 miles as the diameter of the Earth. Calculate the distance Maria is from the horizon if she is standing on a hill that is 125 feet above sea level.
(HINT: 1 mile = 5280 feet)

Maria is approximately 72,290 feet or about 13.7 miles from the horizon.

$$d(h) = \sqrt{h(7918 + h)}$$

$$d(125) = \sqrt{125((7918 \times 5280) + 125)}$$

$$= \sqrt{125(41{,}807{,}040 + 125)}$$

$$= \sqrt{5{,}225{,}895{,}625}$$

$$\approx 72{,}290$$

32. The relationship between the radius of a circle and its area is given by the equation $r = \sqrt{\frac{A}{\pi}}$, where A represents the area of the circle. Write the equation as a function of the area and use 3.14 for π. Calculate the radius of a circle with an area of 50.24 square meters.

The radius of the circle is 4 meters.

$$r(A) = \sqrt{\frac{A}{\pi}}$$

$$r(50.24) = \sqrt{\frac{50.24}{3.14}}$$

$$= \sqrt{16}$$

$$= 4$$

33. The relationship between the side length of a cube and its volume is given by the equation $s = \sqrt[3]{V}$, where s represents the side length and V represents the volume of the cube. Write the equation as a function of the volume. Calculate the side length of a cube that has a volume of 343 cubic inches.

The side length of the cube is 7 inches.

$$s(V) = \sqrt[3]{V}$$

$$= \sqrt[3]{343}$$

$$= 7$$

11

34. The time it takes for an object to fall a certain distance can be calculated using the equation $t = \sqrt{\frac{2d}{g}}$, where d represents distance and g represents the force of gravity on the falling object. Write the equation as a function of the distance and use 9.81 meters per second squared as the force of gravity. Calculate the distance an object will fall in 3 seconds.

The object will fall 44.145 meters in 3 seconds.

$$t(d) = \sqrt{\frac{2d}{9.81}}$$

$$3 = \sqrt{\frac{2d}{9.81}}$$

$$9 = \frac{2d}{9.81}$$

$$88.29 = 2d$$

$$44.145 = d$$

35. The relationship between the radius of a sphere and its surface area is given by the equation $r = \sqrt{\frac{SA}{4\pi}}$, where r represents the radius and SA represents the surface area. Write the equation for the radius as a function of the surface area and use 3.14 for π. Calculate the surface area of a sphere with a 4 foot radius.

The surface area of the sphere is 200.96 square feet.

$$r(SA) = \sqrt{\frac{SA}{12.56}}$$

$$4 = \sqrt{\frac{SA}{12.56}}$$

$$16 = \frac{SA}{12.56}$$

$$200.96 = SA$$

36. The relationship between the side length of the base and the height of a pyramid that is cut out of a cube is given by the equation $s = \sqrt[3]{3V}$, where s represents the length of a side of the base and V represents the volume. Write the equation for the side length as a function of the volume. Calculate the volume of a pyramid with a side length of 4.2 centimeters.

The volume of the pyramid is 24.696 cubic centimeters.

$$s(V) = \sqrt[3]{3V}$$

$$4.2 = \sqrt[3]{3V}$$

$$74.088 = 3V$$

$$24.696 = V$$

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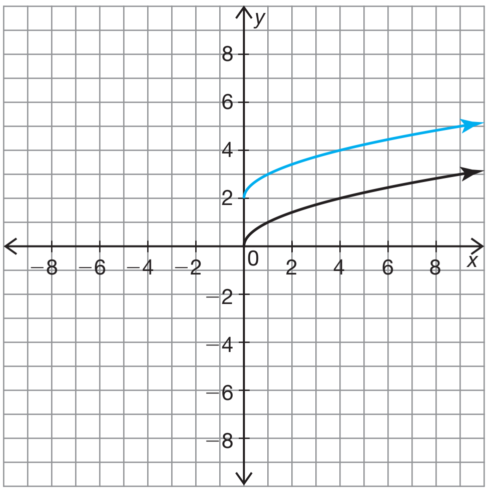
Making Waves

Transformations of Radical Functions

Problem Set

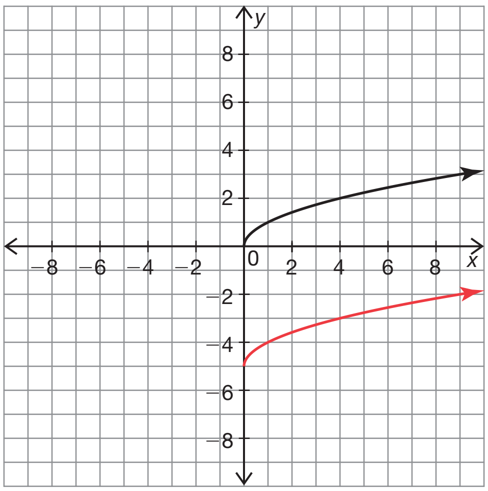
Sketch the graph of the transformation of $f(x) = \sqrt{x}$ as described in each exercise. Write the equation to describe each new function. The graph of $f(x) = \sqrt{x}$ is shown on each grid.

1. Translate the graph up 2 units.



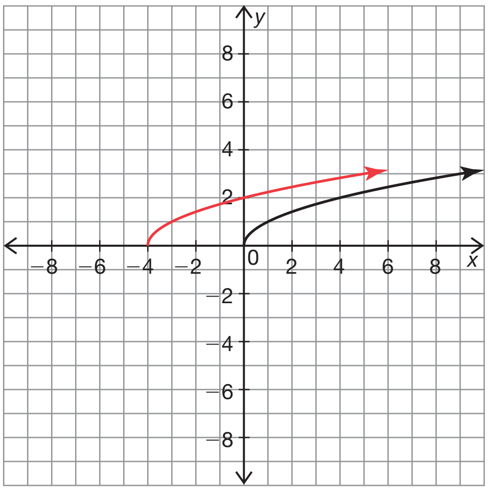
$g(x) = \sqrt{x} + 2$

2. Translate the graph down 5 units.



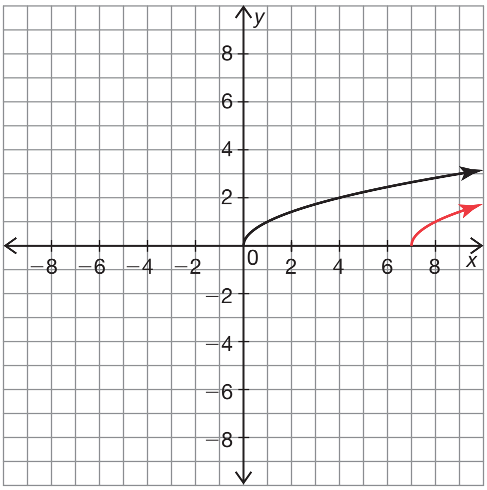
$g(x) = \sqrt{x} - 5$

3. Translate the graph to the left 4 units.



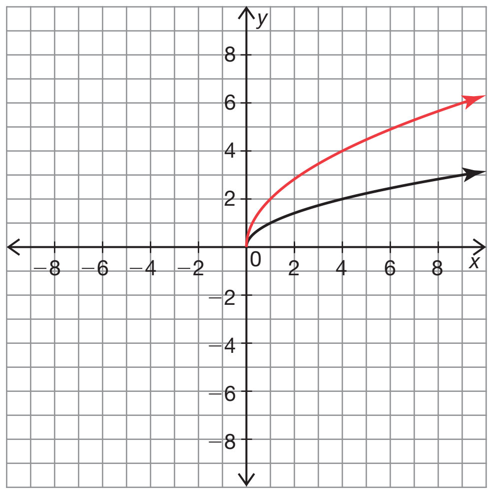
$g(x) = \sqrt{x + 4}$

4. Translate the graph to the right 7 units.



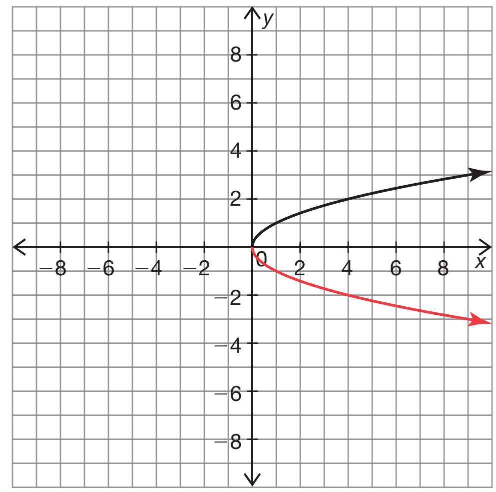
$g(x) = \sqrt{x - 7}$

5. Stretch the graph vertically by a factor of 2.



$g(x) = 2\sqrt{x}$

6. Reflect the graph over the x-axis.

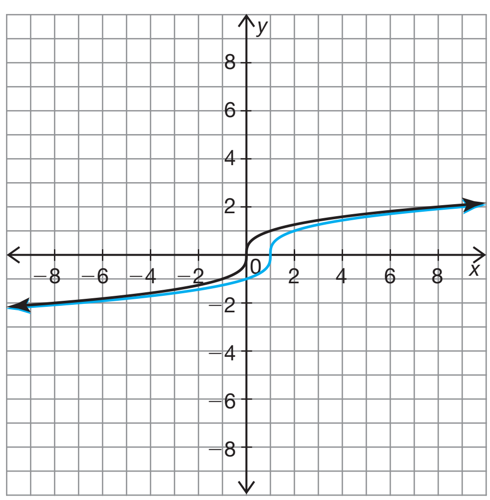


$g(x) = -\sqrt{x}$

11

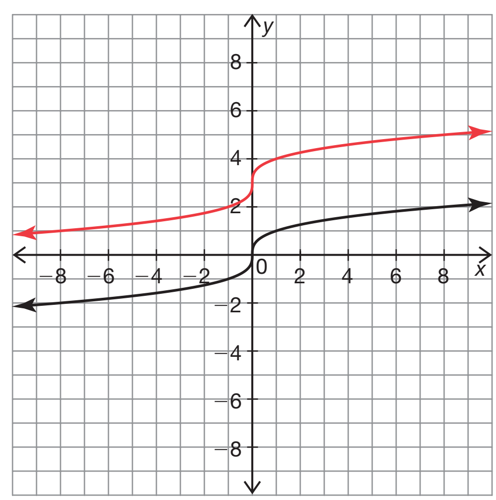
Sketch the graph of the transformation of $f(x) = \sqrt[3]{x}$ as described in each exercise. Write the equation to describe each new function. The graph of $f(x) = \sqrt[3]{x}$ is shown on each grid.

7. Translate the graph to the right 1 unit.



$g(x) = \sqrt[3]{x - 1}$

8. Translate the graph up 3 units.



$g(x) = \sqrt[3]{x} + 3$

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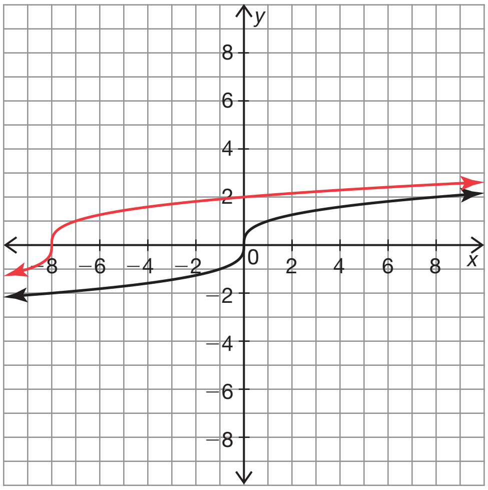
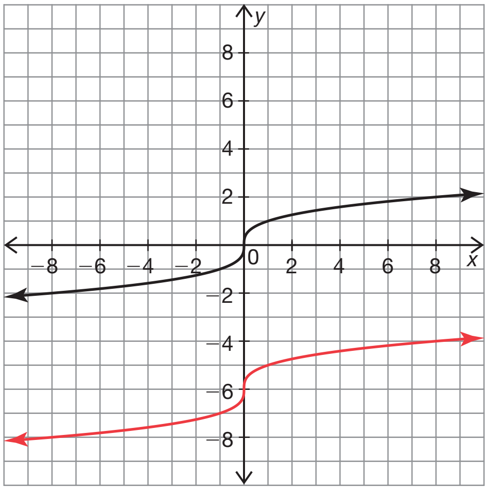
9. Translate the graph to the left 8 units.



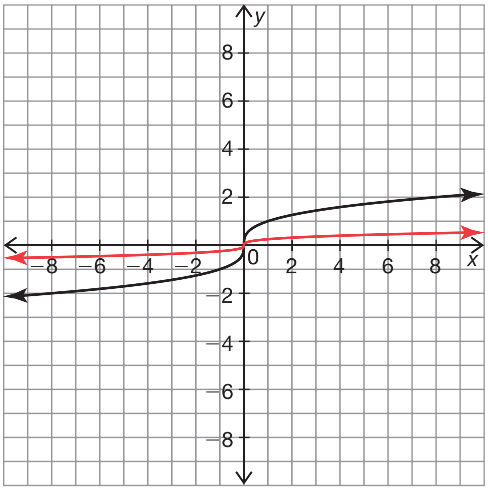
$g(x) = \sqrt[3]{x + 8}$

10. Translate the graph down 6 units.



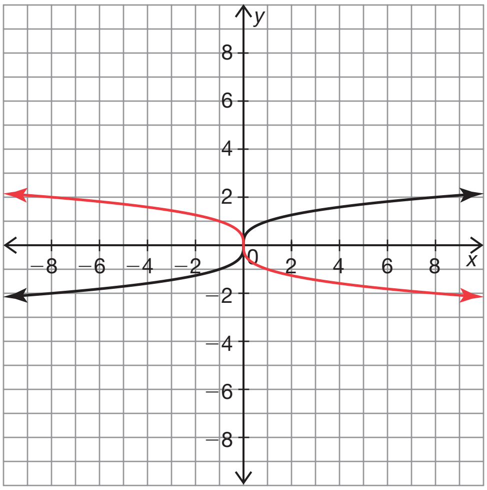
$g(x) = \sqrt[3]{x} - 6$

11. Compress the graph vertically by a factor of $\frac{1}{4}$.



$g(x) = \frac{1}{4}\sqrt[3]{x}$

12. Reflect the graph over the y-axis.



$g(x) = \sqrt[3]{-x}$

Describe how each graph represented by $f(x)$ would be transformed to create the graph represented by $g(x)$.

13. $f(x) = \sqrt{x + 2}$

$g(x) = \sqrt{x + 2} + 5$

The graph of $f(x)$ would be translated up 5 units to create the graph of $g(x)$.

14. $f(x) = \sqrt{x}$

$g(x) = \sqrt{-x}$

The graph of $f(x)$ would be reflected over the y-axis to create the graph of $g(x)$.

15. $f(x) = \sqrt{x - 1}$

$g(x) = 3\sqrt{x - 1}$

The graph of $f(x)$ would be stretched vertically by a factor of 3 to create the graph of $g(x)$.

16. $f(x) = -\sqrt{x} - 4$

$g(x) = -\sqrt{x} + 1$

The graph of $f(x)$ would be shifted up 5 units to create the graph of $g(x)$.

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17. $f(x) = \sqrt[3]{x - 7} + 2$

$g(x) = \sqrt[3]{x - 4} - 3$

The graph of $f(x)$ would be shifted to the left 3 units and down 5 units to create the graph of $g(x)$.

18. $f(x) = \sqrt[3]{x + 6}$

$g(x) = \frac{1}{2}\sqrt[3]{x + 6}$

The graph of $f(x)$ would be compressed vertically by a factor of $\frac{1}{2}$ to create the graph of $g(x)$.

19. $f(x) = \sqrt[3]{x} + 5$

$g(x) = -\sqrt[3]{x} + 5$

The graph of $f(x)$ would be reflected over the y-axis to create the graph of $g(x)$.

20. $f(x) = \sqrt[3]{2x}$

$g(x) = \sqrt[3]{8x}$

The graph of $f(x)$ would be stretched vertically to create the graph of $g(x)$.

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Write an equation for each function by transforming the equation as described.

21. $f(x) = \sqrt{x}$

translated to the right 8 units and up 2 units

$g(x) = \sqrt{x - 8} + 2$

22. $f(x) = \sqrt{2x}$

reflected over the y-axis

$g(x) = \sqrt{-2x}$

23. $f(x) = -\sqrt{x + 4}$

translated to the left 3 units and down 2 units

$g(x) = -\sqrt{x + 7} - 2$

24. $f(x) = \sqrt{x} - 9$

translated to the right 5 units and stretched vertically by a factor of 2

$g(x) = 2\sqrt{(x - 5)} - 9$

25. $f(x) = \sqrt[3]{x}$

translated to the left 6 units and down 3 units

$g(x) = \sqrt[3]{x + 6} - 3$

26. $f(x) = \frac{2}{3}\sqrt[3]{x}$

reflected over the x-axis

$g(x) = -\frac{2}{3}\sqrt[3]{x}$ or $g(x) = \frac{2}{3}\sqrt[3]{-x}$

27. $f(x) = \sqrt[3]{x - 2} + 1$

translated to the right 7 units

$g(x) = \sqrt[3]{x - 9} + 1$

28. $f(x) = -\sqrt[3]{x + 4} - 3$

translated up 7 units and compressed vertically by $\frac{1}{2}$

$g(x) = -\frac{1}{2}\sqrt[3]{x + 4} + 4$

Describe how each transformation changes the domain of the function. In each exercise, $g(x)$ is a transformation of $f(x)$.

29. $f(x) = \sqrt{x}$

$g(x) = \sqrt{x - 2}$

The domain of $f(x)$ is $[0, \infty)$, where as the domain of $g(x)$ is $[2, \infty)$.

30. $f(x) = \sqrt{x - 4}$

$g(x) = \sqrt{4 - x}$

The domain of $f(x)$ is $[4, \infty)$, where as the domain of $g(x)$ is $(-\infty, 4]$.

31. $f(x) = \sqrt{x}$

$g(x) = \sqrt{-x}$

The domain of $f(x)$ is $[0, \infty)$, where as the domain of $g(x)$ is $(-\infty, 0]$.

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32. $f(x) = \sqrt[3]{x}$

$g(x) = \sqrt[3]{x - 3}$

The domain for both functions is $(-\infty, \infty)$.

33. $f(x) = \sqrt[3]{x} + 5$

$g(x) = \sqrt[3]{x} - 5$

The domain for both functions is $(-\infty, \infty)$.

34. $f(x) = \sqrt[3]{x}$

$g(x) = \sqrt[3]{-x}$

The domain for both functions is $(-\infty, \infty)$.

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Keepin' It Real

Extracting Roots and Rewriting Radicals

Problem Set

Rewrite each expression using rational exponents.

1. $\sqrt{x^3y}$

$\sqrt{x^3y} = (x^3y)^{\frac{1}{2}}$

$= x^{\frac{3}{2}}y^{\frac{1}{2}}$

2. $\sqrt[3]{a^2b^4c^5}$

$\sqrt[3]{a^2b^4c^5} = (a^2b^4c^5)^{\frac{1}{3}}$

$= a^{\frac{2}{3}}b^{\frac{4}{3}}c^{\frac{5}{3}}$

3. $\sqrt[4]{f^2g^6}$

$\sqrt[4]{f^2g^6} = (f^2g^6)^{\frac{1}{4}}$

$= f^{\frac{2}{4}}g^{\frac{6}{4}}$

4. $\sqrt[5]{(x + y)^2}$

$\sqrt[5]{(x + y)^2} = [(x + y)^2]^{\frac{1}{5}}$

$= (x + y)^{\frac{2}{5}}$

5. $\sqrt[3]{\frac{r^2s}{t^4}}$

$\sqrt[3]{\frac{r^2s}{t^4}} = \left(\frac{r^2s}{t^4}\right)^{\frac{1}{3}}$

$= \frac{r^{\frac{2}{3}}s^{\frac{1}{3}}}{t^{\frac{4}{3}}}$

6. $\sqrt{a^5b}$

$\sqrt{a^5b} = (a^5b)^{\frac{1}{2}}$

$= a^{\frac{5}{2}}b^{\frac{1}{2}}$

11

7. $\sqrt[4]{\frac{x^2}{y^3}}$

$$\sqrt[4]{\frac{x^2}{y^3}} = \left(\frac{x^2}{y^3}\right)^{\frac{1}{4}}$$
$$= \frac{x^{\frac{2}{4}}}{y^{\frac{3}{4}}}$$
$$= \frac{x^{\frac{1}{2}}}{y^{\frac{3}{4}}}$$

8. $\sqrt[5]{32f^4}$

$$\sqrt[5]{32f^4} = (32f^4)^{\frac{1}{5}}$$
$$= 32^{\frac{1}{5}} \cdot (f^4)^{\frac{1}{5}}$$
$$= 2f^{\frac{4}{5}}$$

Rewrite each expression using radicals.

9. $u^{\frac{2}{3}}w^{\frac{5}{3}}$

$$u^{\frac{2}{3}}w^{\frac{5}{3}} = (u^2w^5)^{\frac{1}{3}}$$
$$= \sqrt[3]{u^2w^5}$$

10. $x^{\frac{1}{2}}y^{\frac{3}{2}}z^{\frac{7}{2}}$

$$x^{\frac{1}{2}}y^{\frac{3}{2}}z^{\frac{7}{2}} = (xy^3z^7)^{\frac{1}{2}}$$
$$= \sqrt{xy^3z^7}$$

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11. $(a + b)^{\frac{3}{4}}$

$$(a + b)^{\frac{3}{4}} = [(a + b)^3]^{\frac{1}{4}}$$
$$= \sqrt[4]{(a + b)^3}$$

12. $f^{\frac{4}{5}}g^{\frac{1}{5}}$

$$f^{\frac{4}{5}}g^{\frac{1}{5}} = (f^4g)^{\frac{1}{5}}$$
$$= \sqrt[5]{f^4g}$$

13. $r^{\frac{1}{2}}s^{\frac{3}{4}}$

$$r^{\frac{1}{2}}s^{\frac{3}{4}} = r^{\frac{2}{4}}s^{\frac{3}{4}}$$
$$= (r^2s^3)^{\frac{1}{4}}$$
$$= \sqrt[4]{r^2s^3}$$

14. $\frac{a^{\frac{3}{2}}b^{\frac{1}{4}}}{c^{\frac{5}{4}}}$

$$\frac{a^{\frac{3}{2}}b^{\frac{1}{4}}}{c^{\frac{5}{4}}} = \frac{a^{\frac{6}{4}}b^{\frac{1}{4}}}{c^{\frac{5}{4}}}$$
$$= \left(\frac{a^6b}{c^5}\right)^{\frac{1}{4}}$$
$$= \sqrt[4]{\frac{a^6b}{c^5}}$$

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15. $x^{\frac{2}{5}}y^{\frac{6}{5}}$

$x^{\frac{2}{5}}y^{\frac{6}{5}} = (x^2y^6)^{\frac{1}{5}}$

$= \sqrt[5]{x^2y^6}$

16. $\dfrac{r^2s^{\frac{2}{3}}}{t^{\frac{1}{3}}u^{\frac{4}{3}}}$

$\dfrac{r^2s^{\frac{2}{3}}}{t^{\frac{1}{3}}u^{\frac{4}{3}}} = \dfrac{r^{\frac{6}{3}}s^{\frac{2}{3}}}{t^{\frac{1}{3}}u^{\frac{4}{3}}}$

$= \left(\dfrac{r^6s^2}{tu^4}\right)^{\frac{1}{3}}$

$= \sqrt[3]{\dfrac{r^6s^2}{tu^4}}$

Simplify each expression.

17. $\sqrt{x^6y^8}$

$\sqrt{x^6y^8} = (x^6y^8)^{\frac{1}{2}}$

$= x^{\frac{6}{2}}y^{\frac{8}{2}}$

$= |x^3|y^4$

18. $\sqrt[3]{a^3b^{12}}$

$\sqrt[3]{a^3b^{12}} = (a^3b^{12})^{\frac{1}{3}}$

$= a^{\frac{3}{3}}b^{\frac{12}{3}}$

$= ab^4$

19. $\sqrt[3]{(x-2)^6}$

$\sqrt[3]{(x-2)^6} = [(x-2)^6]^{\frac{1}{3}}$

$= (x-2)^{\frac{6}{3}}$

$= (x-2)^2$

20. $\sqrt[3]{(5+x)^{12}}$

$\sqrt[3]{(5+x)^{12}} = [(5+x)^{12}]^{\frac{1}{3}}$

$= (5+x)^{\frac{12}{3}}$

$= (5+x)^4$

21. $\sqrt{25y^8}$

$\sqrt{25y^8} = (25y^8)^{\frac{1}{2}}$

$= 25^{\frac{1}{2}} \cdot y^{\frac{8}{2}}$

$= 5y^4$

22. $\sqrt{36z^4}$

$\sqrt{36z^4} = (36z^4)^{\frac{1}{2}}$

$= 36^{\frac{1}{2}} \cdot z^{\frac{4}{2}}$

$= 6z^2$

11

23. $\sqrt{16x^{10}y^8z^2}$

$\sqrt{16x^{10}y^8z^2} = (16x^{10}y^8z^2)^{\frac{1}{2}}$

$= 16^{\frac{1}{2}} \cdot x^{\frac{10}{2}} \cdot y^{\frac{8}{2}} \cdot z^{\frac{2}{2}}$

$= 4|x^5z|y^4$

24. $\sqrt{49x^{12}y^2z^6}$

$\sqrt{49x^{12}y^2z^6} = (49x^{12}y^2z^6)^{\frac{1}{2}}$

$= 49^{\frac{1}{2}} \cdot x^{\frac{12}{2}} \cdot y^{\frac{2}{2}} \cdot z^{\frac{6}{2}}$

$= 7|yz^3|\, x^6$

11

25. $\sqrt[3]{27x^{15}y^9z^3}$

$\sqrt[3]{27x^{15}y^9z^3} = (27x^{15}y^9z^3)^{\frac{1}{3}}$

$= 27^{\frac{1}{3}} \cdot x^{\frac{15}{3}} \cdot y^{\frac{9}{3}} \cdot z^{\frac{3}{3}}$

$= 3x^5y^3z$

26. $\sqrt[4]{16x^{12}y^4z^{16}}$

$\sqrt[4]{16x^{12}y^4z^{16}} = (16x^{12}y^4z^{16})^{\frac{1}{4}}$

$= 16^{\frac{1}{4}} \cdot x^{\frac{12}{4}} \cdot y^{\frac{4}{4}} \cdot z^{\frac{16}{4}}$

$= 2|x^3y|\, z^4$

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Time to Operate!

Multiplying, Dividing, Adding, and Subtracting Radicals

Problem Set

Given variable values greater than zero, perform the indicated operations and extract all roots. Write your final answer in radical form.

1. $\sqrt[4]{a^5b^2} \cdot \sqrt[4]{a^3b^7}$

$$\sqrt[4]{a^5b^2} \cdot \sqrt[4]{a^3b^7} = \sqrt[4]{a^8b^9}$$
$$= \sqrt[4]{a^8 \cdot b^8 \cdot b}$$
$$= a^2b^2\sqrt[4]{b}$$

2. $(2.4\sqrt{2p^5q^9})(-3.1\sqrt{2pq^3})$

$$(2.4\sqrt{2p^5q^9})(-3.1\sqrt{2pq^3}) = -7.44\sqrt{4p^6q^{12}}$$
$$= -7.44\sqrt{2^2 \cdot p^6 \cdot q^{12}}$$
$$= -14.88p^3q^6$$

3. $\sqrt[5]{x^2y^4} \cdot \sqrt[3]{x^3} \cdot \sqrt[5]{y^9}$

$$\sqrt[5]{x^2y^4} \cdot \sqrt[3]{x^3} \cdot \sqrt[5]{y^9} = (x^2y^4)^{\frac{1}{5}} \cdot (x^3)^{\frac{1}{3}} \cdot (y^9)^{\frac{1}{5}}$$
$$= x^{\frac{2}{5}}y^{\frac{4}{5}} \cdot x \cdot y^{\frac{9}{5}}$$
$$= x \cdot x^{\frac{2}{5}} \cdot y^{\frac{13}{5}}$$
$$= x \cdot x^{\frac{2}{5}} \cdot y^{\frac{10}{5}} \cdot y^{\frac{3}{5}}$$
$$= x \cdot x^{\frac{2}{5}} \cdot y^2 \cdot y^{\frac{3}{5}}$$
$$= x \cdot y^2 \cdot (x^2 \cdot y^3)^{\frac{1}{5}}$$
$$= xy^2\sqrt[5]{x^2y^3}$$

4. $\dfrac{\sqrt{r^3t^5}}{\sqrt{rt^4}}$

$$\frac{\sqrt{r^3t^5}}{\sqrt{rt^4}} = \frac{(r^3t^5)^{\frac{1}{2}}}{(rt^4)^{\frac{1}{2}}}$$
$$= \frac{r^{\frac{3}{2}}t^{\frac{5}{2}}}{r^{\frac{1}{2}}t^{\frac{4}{2}}}$$
$$= rt^{\frac{1}{2}}$$
$$= r\sqrt{t}$$

11

5. $-\sqrt{27s^5t^8} \cdot \sqrt{2st^3} \cdot \sqrt[3]{s^6t^9}$

$$-\sqrt{27s^5t^8} \cdot \sqrt{2st^3} \cdot \sqrt[3]{s^6t^9} = -\sqrt{54s^6t^{11}} \cdot \sqrt[3]{s^6t^9}$$
$$= -\left(54^{\frac{1}{2}}s^3t^{\frac{11}{2}}\right) \cdot (s^2t^3)$$
$$= -\left(9^{\frac{1}{2}} \cdot 6^{\frac{1}{2}} \cdot s^5 \cdot t^{\frac{17}{2}}\right)$$
$$= -\left(3 \cdot 6^{\frac{1}{2}} \cdot s^5 \cdot t^{\frac{16}{2}} \cdot t^{\frac{1}{2}}\right)$$
$$= -3s^5t^8\sqrt{6t}$$

6. $\dfrac{-7\sqrt[4]{x}}{5\sqrt[3]{x}}$

$$\frac{-7\sqrt[4]{x}}{5\sqrt[3]{x}} = \frac{-7 \cdot x^{\frac{1}{4}}}{5 \cdot x^{\frac{1}{3}}}$$
$$= -7 \cdot x^{\frac{1}{4}} \cdot \frac{1}{5} \cdot x^{-\frac{1}{3}}$$
$$= -\frac{7}{5} \cdot x^{-\frac{1}{12}}$$
$$= \frac{-7 \cdot x^{\frac{11}{12}}}{5 \cdot x^{\frac{1}{12}} \cdot x^{\frac{11}{12}}}$$
$$= \frac{-7 \cdot x^{\frac{11}{12}}}{5 \cdot x}$$
$$= \frac{-7\sqrt[12]{x^{11}}}{5x}$$

7. $\dfrac{\sqrt[3]{4096x^5y^8z^2}}{\sqrt[3]{8x^8y}}$

$$\frac{\sqrt[3]{4096x^5y^8z^2}}{\sqrt[3]{8x^8y}} = \frac{(4096x^5y^8z^2)^{\frac{1}{3}}}{(8x^8y)^{\frac{1}{3}}}$$
$$= \frac{16 \cdot x^{\frac{5}{3}} \cdot y^{\frac{8}{3}} \cdot z^{\frac{2}{3}}}{2 \cdot x^{\frac{8}{3}} \cdot y^{\frac{1}{3}}}$$
$$= \frac{8 \cdot y^{\frac{7}{3}} \cdot z^{\frac{2}{3}}}{x}$$
$$= \frac{8 \cdot y^{\frac{6}{3}} \cdot y^{\frac{1}{3}} \cdot z^{\frac{2}{3}}}{x}$$
$$= \frac{8y^2\sqrt[3]{yz^2}}{x}$$

8. $\dfrac{9.8\sqrt{a^3b^4}}{\sqrt[4]{16a^8b^6}}$

$$\frac{9.8\sqrt{a^3b^4}}{\sqrt[4]{16a^8b^6}} = \frac{9.8 \cdot (a^3b^4)^{\frac{1}{2}}}{(16a^8b^6)^{\frac{1}{4}}}$$
$$= \frac{9.8 \cdot a^{\frac{3}{2}} \cdot b^2}{2 \cdot a^2 \cdot b^{\frac{3}{2}}}$$
$$= 4.9 \cdot a^{-\frac{1}{2}} \cdot b^{\frac{1}{2}}$$
$$= \frac{4.9 \cdot b^{\frac{1}{2}}}{a^{\frac{1}{2}}}$$
$$= \frac{4.9 \cdot b^{\frac{1}{2}} \cdot a^{\frac{1}{2}}}{a^{\frac{1}{2}} \cdot a^{\frac{1}{2}}}$$
$$= \frac{4.9\sqrt{ab}}{a}$$

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Given variable values greater than zero, perform the indicated operations and extract all roots. Write your final answer in radical form.

9. $3\sqrt[4]{w} + 5\sqrt[4]{w}$

$3\sqrt[4]{w} + 5\sqrt[4]{w} = 8\sqrt[4]{w}$

10. $\sqrt{x^3y^6} + 7\sqrt{x^3y^6} + 2\sqrt{x^2y^4}$

$\sqrt{x^3y^6} + 7\sqrt{x^3y^6} + 2\sqrt{x^2y^4} = 8\sqrt{x^3y^6} + 2\sqrt{x^2y^4}$

$= 8x^{\frac{3}{2}}y^3 + 2xy^2$

$= 8xy^3\sqrt{x} + 2xy^2$

11. $2.6\sqrt[3]{p^5q} + 3.2\sqrt[3]{p^5q}$

$2.6\sqrt[3]{p^5q} + 3.2\sqrt[3]{p^5q} = 5.8\sqrt[3]{p^5q}$

$= 5.8\sqrt[3]{p^3p^2q}$

$= 5.8p\sqrt[3]{p^2q}$

12. $\frac{\sqrt[5]{a^3b^7c^2}}{10} + \frac{3\sqrt[5]{a^3b^7c^2}}{10}$

$\frac{\sqrt[5]{a^3b^7c^2}}{10} + \frac{3\sqrt[5]{a^3b^7c^2}}{10} = \frac{4\sqrt[5]{a^3b^7c^2}}{10}$

$= \frac{4\sqrt[5]{a^3b^5b^2c^2}}{10}$

$= \frac{2b\sqrt[5]{a^3b^2c^2}}{5}$

13. $11\sqrt[3]{x} - 5\sqrt[3]{x}$

$11\sqrt[3]{x} - 5\sqrt[3]{x} = 6\sqrt[3]{x}$

14. $8.2\sqrt[3]{c^6d^9} - 6.5\sqrt[5]{c^{10}d^{15}}$

$8.2\sqrt[3]{c^6d^9} - 6.5\sqrt[5]{c^{10}d^{15}}$

$= (8.2 \cdot c^2 \cdot d^3) - (6.5 \cdot c^2 \cdot d^3)$

$= 1.7c^2d^3$

15. $2\sqrt{a^9b^5} - 5\sqrt{a^9b^5}$

$$2\sqrt{a^9b^5} - 5\sqrt{a^9b^5} = -3\sqrt{a^9b^5}$$
$$= -3\sqrt{a^8 \cdot a \cdot b^4 \cdot b}$$
$$= -3a^4b^2\sqrt{ab}$$

16. $9\sqrt[3]{r^4s^3} - 2\sqrt{r^4s^3} - 3\sqrt[3]{r^4s^3}$

$$9\sqrt[3]{r^4s^3} - 2\sqrt{r^4s^3} - 3\sqrt[3]{r^4s^3}$$
$$= 6\sqrt[3]{r^4s^3} - 2\sqrt{r^4s^3}$$
$$= \left(6 \cdot r^{\frac{4}{3}} \cdot s\right) - \left(2 \cdot r^2 \cdot s^{\frac{3}{2}}\right)$$
$$= \left(6 \cdot r^{\frac{3}{3}} \cdot r^{\frac{1}{3}} \cdot s\right) - \left(2 \cdot r^2 \cdot s^{\frac{2}{2}} \cdot s^{\frac{1}{2}}\right)$$
$$= 6rs\sqrt[3]{r} - 2r^2s\sqrt{s}$$

Given variable values greater than zero, perform the indicated operations and extract all roots. Write your final answer in radical form.

17. $9\sqrt{y}(5\sqrt{y} - \sqrt{y})$

$$9\sqrt{y}(5\sqrt{y} - \sqrt{y}) = 45\sqrt{y^2} - 9\sqrt{y^2}$$
$$= 45y - 9y$$
$$= 36y$$

18. $a^2 \cdot (2\sqrt{b})^4$

$$a^2 \cdot (2\sqrt{b})^4 = a^2 \cdot 2^4 \cdot b^2$$
$$= 16a^2b^2$$

11

19. $-5\sqrt[3]{x}\left(\sqrt[3]{x^6} + 2x^3\right)$

$$-5\sqrt[3]{x}\left(\sqrt[3]{x^6} + 2x^3\right) = -5\sqrt[3]{x^7} + \left(-10x^3\sqrt[3]{x}\right)$$
$$= -5\sqrt[3]{x^6x} + \left(-10x^3\sqrt[3]{x}\right)$$
$$= -5x^2\sqrt[3]{x} - 10x^3\sqrt[3]{x}$$

20. $\dfrac{\left(3\sqrt{x^7y^4}\right)\left(\sqrt[3]{-8x^6y^4}\right)}{\sqrt{9x^3y^{10}}}$

$$\frac{\left(3\sqrt{x^7y^4}\right)\left(\sqrt[3]{-8x^6y^4}\right)}{\sqrt{9x^3y^{10}}} = \frac{\left(3x^{\frac{7}{2}}y^2\right)\left(-2x^2y^{\frac{4}{3}}\right)}{3x^{\frac{3}{2}}y^5}$$
$$= \frac{-6x^{\frac{11}{2}}y^{\frac{10}{3}}}{3x^{\frac{3}{2}}y^5}$$
$$= \frac{-2x^4}{y^{\frac{5}{3}}}$$
$$= \frac{-2x^4}{y^{\frac{3}{3}}y^{\frac{2}{3}}}$$
$$= \frac{-2x^4}{y\sqrt[3]{y^2}}$$

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21. $\sqrt{2x^5}\left(\sqrt{3xy} - 5\sqrt[3]{2x^5}\right)$

$\sqrt{2x^5}\left(\sqrt{3xy} - 5\sqrt[3]{2x^5}\right)$

$= \sqrt{6x^6y} - \left(\sqrt{2x^5}\right)\left(5\sqrt[3]{2x^5}\right)$

$= \left(6^{\frac{1}{2}} \cdot x^3 \cdot y^{\frac{1}{2}}\right) - \left(2^{\frac{1}{2}} \cdot x^{\frac{5}{2}}\right)\left(5 \cdot 2^{\frac{1}{3}} \cdot x^{\frac{5}{3}}\right)$

$= x^3\sqrt{6y} - \left(5 \cdot 2^{\frac{5}{6}} \cdot x^{\frac{25}{6}}\right)$

$= x^3\sqrt{6y} - 5x^4\sqrt[6]{32x}$

22. $\dfrac{6\sqrt[3]{p^2}\left(2\sqrt[3]{p^4} + 4p\sqrt[3]{p^4}\right)}{3\sqrt[3]{p^9}}$

$\dfrac{6\sqrt[3]{p^2}\left(2\sqrt[3]{p^4} + 4p\sqrt[3]{p^4}\right)}{3\sqrt[3]{p^9}} = \dfrac{12\sqrt[3]{p^6} + 24p\sqrt[3]{p^6}}{3\sqrt[3]{p^9}}$

$= \dfrac{12p^2 + 24p^2}{3p^3}$

$= \dfrac{36p^2}{3p^3}$

$= \dfrac{12}{p}$

23. $5a\sqrt{y^2} - 4\sqrt[5]{a^5y^5} + 6y\sqrt[3]{a^3}$

$5a\sqrt{y^2} - 4\sqrt[5]{a^5y^5} + 6y\sqrt[3]{a^3} = 5ay - 4ay + 6ay$

$= 7ay$

24. $\dfrac{6\sqrt[4]{a^3}}{\sqrt[4]{3a^7}} + \dfrac{2\sqrt[5]{a}}{\sqrt[5]{a^6}}$

$\dfrac{6\sqrt[4]{a^3}}{\sqrt[4]{3a^7}} + \dfrac{2\sqrt[5]{a}}{\sqrt[5]{a^6}} = \dfrac{6a^{\frac{3}{4}}}{3^{\frac{1}{4}}a^{\frac{7}{4}}} + \dfrac{2a^{\frac{1}{5}}}{a^{\frac{6}{5}}}$

$= \dfrac{2 \cdot 3 \cdot a^{\frac{3}{4}}}{3^{\frac{1}{4}} \cdot a^{\frac{7}{4}}} + \dfrac{2 \cdot a^{\frac{1}{5}}}{a^{\frac{6}{5}}}$

$= \dfrac{2 \cdot 3^{\frac{3}{4}}}{a} + \dfrac{2}{a}$

$= \dfrac{2\sqrt[4]{27} + 2}{a}$

11

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Look to the Horizon
Solving Radical Equations

Problem Set

Solve each equation. Check for extraneous solutions.

1. $\sqrt{3x} = 6$

$(\sqrt{3x})^2 = (6)^2$

$3x = 36$

$x = 12$

Check:

$\sqrt{3(12)} \stackrel{?}{=} 6$

$\sqrt{36} \stackrel{?}{=} 6$

$6 = 6$ ✓

Solution: $x = 12$

2. $\sqrt{4x} = 8$

$(\sqrt{4x})^2 = (8)^2$

$4x = 64$

$x = 16$

Check:

$\sqrt{4(16)} \stackrel{?}{=} 8$

$\sqrt{64} \stackrel{?}{=} 8$

$8 = 8$ ✓

Solution: $x = 16$

3. $\sqrt[4]{5x - 1} = 2$

$(\sqrt[4]{5x - 1})^4 = (2)^4$

$5x - 1 = 16$

$5x = 17$

$x = \frac{17}{5}$

Check:

$\sqrt[4]{5\left(\frac{17}{5}\right) - 1} \stackrel{?}{=} 2$

$\sqrt[4]{16} \stackrel{?}{=} 2$

$2 = 2$

Solution: $x = \frac{17}{5}$ ✓

4. $\sqrt[5]{3x - 3} = 2$

$(\sqrt[5]{3x - 3})^5 = (2)^5$

$3x - 3 = 32$

$3x = 35$

$x = \frac{35}{3}$

Check:

$\sqrt[5]{3\left(\frac{35}{3}\right) - 3} \stackrel{?}{=} 2$

$\sqrt[5]{32} \stackrel{?}{=} 2$

$2 = 2$

Solution: $x = \frac{35}{3}$ ✓

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5. $2\sqrt[3]{x} + 5 = 1$

$2\sqrt[3]{x} = -4$

$\sqrt[3]{x} = -2$

$(\sqrt[3]{x})^3 = (-2)^3$

$x = -8$

Check:

$2\sqrt[3]{-8} + 5 \stackrel{?}{=} 1$

$-4 + 5 \stackrel{?}{=} 1$

$1 = 1$ ✓

Solution: $x = -8$

6. $4\sqrt[5]{x} + 5 = -3$

$4\sqrt[5]{x} = -8$

$\sqrt[5]{x} = -2$

$(\sqrt[5]{x})^5 = (-2)^5$

$x = -32$

Check:

$4\sqrt[5]{-32} + 5 \stackrel{?}{=} -3$

$-8 + 5 \stackrel{?}{=} -3$

$-3 = -3$ ✓

Solution: $x = -3$

7. $\sqrt{10x - 1} - 7 = -5$

$\sqrt{10x - 1} = 2$

$\left(\sqrt{10x - 1}\right)^2 = (2)^2$

$10x - 1 = 4$

$10x = 5$

$x = \frac{1}{2}$

Check:

$\sqrt{10\left(\frac{1}{2}\right) - 1} - 7 \stackrel{?}{=} -5$

$\sqrt{4} - 7 \stackrel{?}{=} -5$

$-5 = -5$ ✓

Solution: $x = \frac{1}{2}$

8. $\sqrt{9x + 3} - 11 = -8$

$\sqrt{9x + 3} = 3$

$\left(\sqrt{9x + 3}\right)^2 = (3)^2$

$9x + 3 = 9$

$9x = 6$

$x = \frac{2}{3}$

Check:

$\sqrt{9\left(\frac{2}{3}\right) + 3} - 11 \stackrel{?}{=} -8$

$\sqrt{9} - 11 \stackrel{?}{=} -8$

$-8 = -8$ ✓

Solution: $x = \frac{2}{3}$

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Solve each equation. Check for extraneous solutions.

9. $3 + x = \sqrt{4x + 9}$

$x^2 + 6x + 9 = 4x + 9$

$x^2 + 2x = 0$

$x(x + 2) = 0$

$x = 0, x = -2$

Solution: $x = 0$ or $x = -2$

$3 + (0) \stackrel{?}{=} \sqrt{4(0) + 9}$

$3 \stackrel{?}{=} \sqrt{9}$

$3 = 3$ ✓

$3 + (-2) \stackrel{?}{=} \sqrt{4(-2) + 9}$

$1 \stackrel{?}{=} \sqrt{1}$

$1 = 1$ ✓

10. $x - 4 = \sqrt{2x - 9}$

$x^2 - 8x + 16 = 2x - 9$

$x^2 - 10x + 25 = 0$

$(x - 5)^2 = 0$

$x - 5 = 0$

$x = 5$

Solution: $x = 5$

$(5) - 4 \stackrel{?}{=} \sqrt{2(5) - 9}$

$1 \stackrel{?}{=} \sqrt{1}$

$1 = 1$ ✓

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11. $2x - 2 = \sqrt{x + 2}$

$4x^2 - 8x + 4 = x + 2$

$4x^2 - 9x + 2 = 0$

$(4x - 1)(x - 2) = 0$

$x = \frac{1}{4}, x = 2$

Solution: $x = 2$

$2\left(\frac{1}{4}\right) - 2 \stackrel{?}{=} \sqrt{\left(\frac{1}{4}\right) + 2}$

$-\frac{3}{2} \stackrel{?}{=} \sqrt{\frac{9}{4}}$

$-\frac{3}{2} \neq \frac{3}{2}$

Extraneous solution

$2(2) - 2 \stackrel{?}{=} \sqrt{(2) + 2}$

$2 \stackrel{?}{=} \sqrt{4}$

$2 = 2$ ✓

12. $x + 2 = \sqrt{3x + 10}$

$x^2 + 4x + 4 = 3x + 10$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3, x = 2$

Solution: $x = 2$

$(-3) + 2 \stackrel{?}{=} \sqrt{3(-3) + 10}$

$-1 \stackrel{?}{=} \sqrt{1}$

$-1 \neq 1$

Extraneous Solution

$(2) + 2 \stackrel{?}{=} \sqrt{3(2) + 10}$

$4 \stackrel{?}{=} \sqrt{16}$

$4 = 4$ ✓

13. $x = \sqrt[3]{2x^2 + 8x}$

$x^3 = 2x^2 + 8x$

$x^3 - 2x^2 - 8x = 0$

$x(x^2 - 2x - 8) = 0$

$x(x - 4)(x + 2) = 0$

$x = -2, x = 0, x = 4$

Solution: $x = -2$ or $x = 0$ or $x = 4$

$(-2) \stackrel{?}{=} \sqrt[3]{2(-2)^2 + 8(-2)}$

$-2 \stackrel{?}{=} \sqrt[3]{-8}$

$-2 = -2$ ✓

$(0) \stackrel{?}{=} \sqrt[3]{2(0)^2 + 8(0)}$

$0 \stackrel{?}{=} \sqrt[3]{0}$

$0 = 0$ ✓

$(4) \stackrel{?}{=} \sqrt[3]{2(4)^2 + 8(4)}$

$4 \stackrel{?}{=} \sqrt[3]{64}$

$4 = 4$ ✓

14. $-x = \sqrt[3]{x^2 - 12x}$

$-x^3 = x^2 - 12x$

$0 = x^3 + x^2 - 12x$

$0 = x(x^2 + x - 12)$

$0 = x(x + 4)(x - 3)$

$x = -4, x = 0, x = 3$

Solution: $x = -4$ or $x = 0$ or $x = 3$

$-(-4) \stackrel{?}{=} \sqrt[3]{(-4)^2 - 12(-4)}$

$4 \stackrel{?}{=} \sqrt[3]{64}$

$4 = 4$ ✓

$-(-0) \stackrel{?}{=} \sqrt[3]{(0)^2 - 12(0)}$

$0 \stackrel{?}{=} \sqrt[3]{0}$

$0 = 0$ ✓

$-(3) \stackrel{?}{=} \sqrt[3]{(3)^2 - 12(3)}$

$-3 \stackrel{?}{=} \sqrt[3]{-27}$

$-3 = -3$ ✓

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15. $\sqrt{3x - 5} = 1 - \sqrt{2x}$

$\sqrt{3x - 5} = 1 - \sqrt{2x}$

$3x - 5 = 1 - 2\sqrt{2x} + 2x$

$x - 6 = -2\sqrt{2x}$

$x^2 - 12x + 36 = 8x$

$x^2 - 20x + 36 = 0$

$(x - 2)(x - 18) = 0$

$x = 2, x = 18$

$\sqrt{3(2) - 5} \stackrel{?}{=} 1 - \sqrt{2(2)}$

$\sqrt{1} \stackrel{?}{=} 1 - 2$

$1 \neq -1$

Extraneous solution

$\sqrt{3(18) - 5} \stackrel{?}{=} 1 - \sqrt{2(18)}$

$\sqrt{49} \stackrel{?}{=} 1 - 6$

$7 \neq -5$

Extraneous solution

16. $\sqrt{x + 1} = \sqrt{2x + 1} + 2$

$\sqrt{x + 1} = \sqrt{2x + 1} + 2$

$x + 1 = 2x + 1 + 4\sqrt{2x + 1} + 4$

$-x - 4 = 4\sqrt{2x + 1}$

$x^2 + 8x + 16 = 16(2x + 1)$

$x^2 + 8x + 16 = 32x + 16$

$x^2 + 8x = 32x$

$x^2 - 24x = 0$

$x(x - 24) = 0$

$x = 0, x = 24$

$\sqrt{0 + 1} \stackrel{?}{=} \sqrt{2(0) + 1} + 2$

$\sqrt{1} \stackrel{?}{=} \sqrt{1} + 2$

$1 \neq 3$

Extraneous solution

$\sqrt{24 + 1} \stackrel{?}{=} \sqrt{2(24) + 1} + 2$

$\sqrt{25} \stackrel{?}{=} \sqrt{49} + 2$

$5 \neq 9$

Extraneous solution

Solve each problem. Check for extraneous solutions.

17. The distance between any two points on a coordinate grid, d, can be calculated by using the equation $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where (x_1, y_1) represent the coordinates of one point and (x_2, y_2) represent the coordinates of the other point. Identify the point(s) on the x-axis $(x, 0)$, that is (are) exactly 8 units from the point $(2, -3)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(8) = \sqrt{(x - 2)^2 + (0 - (-3))^2}$

$64 = x^2 - 4x + 4 + 9$

$0 = x^2 - 4x - 51$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-51)}}{2(1)} = \dfrac{4 \pm 2\sqrt{55}}{2} = 2 \pm \sqrt{55}$

$(2 + \sqrt{55}, 0)$ and $(2 - \sqrt{55}, 0)$

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18. The distance between any two points on a coordinate grid, d, can be calculated by using the equation $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where (x_1, y_1) represent the coordinates of one point and (x_2, y_2) represent the coordinates of the other point. Identify the point(s) on the y-axis $(0, y)$, that is (are) exactly 5 units from the point $(-3, -4)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$(5) = \sqrt{(0 - (-3))^2 + (y - (-4))^2}$

$25 = 9 + y^2 + 8y + 16$

$0 = y^2 + 8y$

$0 = y(y + 8)$

$y = 0, y = -8$

$(0, 0)$ and $(0, -8)$

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19. The radius of a circle on a coordinate grid that is centered at the origin, r, can be calculated by using the equation $r = \sqrt{x^2 + y^2}$, where x represents the x-coordinate and y represents the y-coordinate of a point on the circle. Determine the x-coordinate(s) of a point(s) $(x, 6)$ on a circle with a radius of 8.

$r = \sqrt{x^2 + y^2}$

$8 = \sqrt{x^2 + 6^2}$

$64 = x^2 + 36$

$28 = x^2$

$\pm 2\sqrt{7} = x$

$(-2\sqrt{7}, 0)$ and $(2\sqrt{7}, 0)$

11

20. The minute you drive a newly purchased car off the lot, its resale value drops immediately. The equation $r = 1 - \sqrt[3]{\frac{v}{c}}$ models a car's immediate resale value, where v represents the immediate resale value of the car, c represents the original cost of the car, and r represents the depreciation rate. Determine the immediate resale value of the car if the original cost was $29,500 and the depreciation rate is 7%. Round your answer to the nearest cent.

$r_d = 1 - \sqrt[3]{\frac{v}{c}}$

$0.07 = 1 - \sqrt[3]{\frac{v}{29{,}500}}$

$-0.93 = -\sqrt[3]{\frac{v}{29{,}500}}$

$0.804357 = \frac{v}{29{,}500}$

$23{,}728.53 \approx v$

The current value of the car is $23,728.53.

21. The speed, in meters per second, of a tsunami can be determined by using the formula $s = \sqrt{9.8d}$, where d is the depth of the ocean in meters. Suppose a tsunami is traveling at a speed of 8.3 kilometers per second. How deep is the ocean at that point? (HINT: 1 kilometer = 1000 meters)

$$s = \sqrt{9.8d}$$

$$8300 = \sqrt{9.8d}$$

$$68{,}890{,}000 = 9.8d$$

$$7{,}029{,}591.837 = d$$

The depth of the ocean is 7,029,591.837 meters or about 7,030 kilometers.

22. Melissa deposited $2580 in an account 3 years ago. The interest is compounded once a year, and the equation $r = \sqrt[3]{\frac{A}{2580}} - 1$, where A is the current balance, can be used to calculate the interest rate. If the interest rate is 3.5%, how much does Melissa currently have in her account? Round your answer to the nearest cent. (HINT: Write the interest rate as its decimal equivalent before substituting it into the equation.)

$$r = \sqrt[3]{\frac{A}{2580}} - 1$$

$$0.035 = \sqrt[3]{\frac{A}{2580}} - 1$$

$$1.035 = \sqrt[3]{\frac{A}{2580}}$$

$$1.1087 \approx \frac{A}{2580}$$

$$2860.49 \approx A$$

The current balance is approximately $2860.49.